

## DERIVATION OF MATHEMATICAL MODEL OF MECHANISM FOR DRIVE OF TRANSPORT MEANS ADDITIONAL DEVICE

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**Abstract.** Simulations and computation are widely used in analysing and investigation of operational properties of mechanisms. Such mechanisms can be placed on transport means and they usually serve to drive additional devices. Additional devices are inseparable part of lorries, buses as well as railway vehicles, such as locomotives with independent traction. These additional devices serve for drive of compressors or other similar devices. The article is aimed at derivation of a mathematical model of a drive mechanism for a drive of an additional device of a vehicle. It is a drive mechanism, which includes several wheels and a belt. There are derived equations of motions of a drive mechanism with a rigid belt and with a flexible belt. The presented mathematical models are derived by means of the Lagrange's equations of the second kind method. After finding the required energies, the method of the reduction of mass and force quantities to the chosen part is applied. In this case, it is reduced to the drive shaft of the considered drive mechanism. The presented mechanism with the rigid belt is described by means of one equation of motion and with the flexible belt with two equations of motion. These equations of motion are solved by means of the Matlab software. The last part of the article includes illustrations of the results in a form of graphical outputs. The presented research and the solved mechanism are a basis of creation of a multibody model. It was proven that the flexibility of the belt leads to relative deflections of the drive and driven part of the mechanism.

**Keywords:** drive mechanism, mathematical model, equation of motion, Matlab, transport mean.

### Introduction

Generally, transport means are equipped by additional accessories. Even, it is possible to declare, that they are an inseparable part of transport means. Recently, it is not possible that various kinds of transport means can reliably work without them. Additional accessories are applied for many kinds of working processes. In case of lorries, busses or railway vehicles with an independent powertrain traction (or also with a dependent traction system), a combustion engine [1; 2] or an electromotor is connected with a compressor as a source of compressed air. Surely, there are also other types of additional accessories, which are powered by an engine or an electromotor [3-7]. Usually, a mechanical drive of additional accessories consists of a belt, which is guided in wheels. Depending on a kind of the belt, which can be a V-belt, a toothed belt, or a flat belt, the mechanical drive mechanism includes a V-pulley, a toothed pulley or a flat pulley [8]. These drive mechanisms include a rigid (pulleys) and flexible (belts) components, which move during their operation. In principle, they perform a kind of rotational movement, and their rotational speed is considerable. From the dynamics point of view, the dynamical effects arising during the operation of mechanical drive systems of additional accessories can lead to oscillation of these systems, which can result to damage or even destroying structural units of transport means. Therefore, it is worth to investigate the behaviour of these mechanical drive systems regarding to their dynamical properties [9-12].

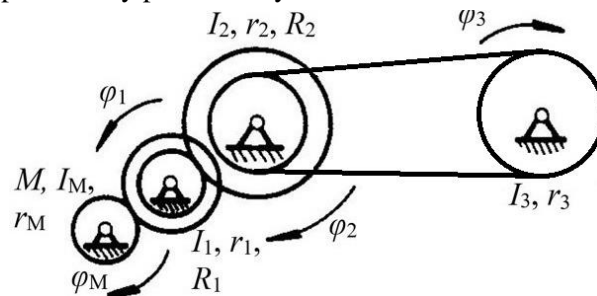
The goal of the presented research is derivation of a mathematical model of a drive mechanism of additional accessories of a vehicle. Such mathematical model can be applied for investigation and evaluation of dynamic effects, which relate with additional accessories operation. A presented mathematical model is derived by means of the Lagrange's equations of the second kind method [13, 14]. After that, the method of reduction of mass (inertia) parameters and forces (torques) was applied.

The mechanical drive system itself consists of several components. In the reality, there are many kinds of such mechanisms, which include more or less components. The presented research is focused on chosen mechanisms with rigid and flexible elements. The system includes drive components and driven components. The systems are reduced to the drive component of the mechanical drive system. This model is solved by means of the Matlab software [15-17]. The solution consists in writing the mathematical model to the Matlab software in a form of script file and the results are obtained in the form of numerical values (matrices and vectors). Then, these numerical values are converted to the graphical outputs, which are clearer and better to understand.

**Materials and methods**

A suitable dynamical model is indispensable for derivation of a mathematical model of a mechanical drive system of additional accessories. Therefore, it is necessary to create such a dynamical model, which includes all essential components and properties of the solved mechanism. The process of finding the wanted solution includes several steps.

As it is described above, a proper dynamical model is set-up. The dynamical model of the considered mechanical drive system is depicted in Fig. 1. It should be mentioned that it is a principle scheme and it does not represent any particular system.



**Fig. 1. Scheme of a dynamical model of a mechanical drive system with a rigid belt**

As it can be seen, this system includes four bodies, which are marked as *M*, 1, 2, 3. These rigid bodies are wheels. Wheels *M* and 1 are toothed wheels and wheels 2 and L are designed for mounting a belt. The wheel *M* is a drive wheel, and it is considered that the load of the system acts on the wheel 3. These four wheels include mass. It means, that their dynamical properties are characterized by their moments of inertia regarding to the axis of rotation (axes of rotation are heading to the basic plane of the scheme in Fig. 1). The belt is considered as massless.

A geometry of the wheel 2 is designed to be a gear pair with the wheel *M* as well as with the wheel 2. The loaded wheel (wheel 3) is driven by the massless belt.

Application of the Lagrange’s equations of the second kind method needs to know energies of the solved mechanical system, as it can be seen in the general form of this method:

$$\frac{d}{dt} \left( \frac{\partial D_K}{\partial \dot{q}_i} \right) - \frac{\partial D_K}{\partial q_i} + \frac{\partial D_D}{\partial \dot{q}_i} + \frac{\partial D_P}{\partial q_i} = Q_i, \tag{1}$$

- where  $D_K$  – kinetic energy, J;
- $D_D$  – dissipative energy, J;
- $D_P$  – potential energy, J;
- $\dot{q}_i$  – generalized velocities (in our case angular velocities),  $\text{rad} \cdot \text{s}^{-1}$ ;
- $q_i$  – generalized coordinates (in our case angular deflection), rad;
- $Q_i$  – external loads of the system (in our case torques),  $\text{N} \cdot \text{m} \cdot \text{rad}^{-1}$ .

As it is known, kinetic energy of rotating bodies is calculated based on the formula:

$$D_k = \frac{1}{2} \cdot I_i \cdot \dot{\varphi}_i^2, \tag{2}$$

- where  $I_i$  – moments of inertia of rotating bodies,  $\text{kg} \cdot \text{m}^2$ ;
- $\dot{\varphi}_i$  – angular velocities of rotating bodies,  $\text{rad} \cdot \text{s}^{-1}$ .

In our solved case, potential energy of the system equals to zero ( $D_P = 0$ ), because all bodies are rigid, and the system does not change its vertical position in the gravitational field. Moreover, the system is simplified by the fact, that neither friction nor damping component is considered in the system. Thus, dissipative energy also equals to zero ( $D_D = 0$ ).

A generalized coordinate is chosen as an angular deflection  $\varphi_M$  of the drive wheel *M*.

Now, kinetic energy of the solved mechanical system is in the general form as following:

$$D_K = \frac{1}{2} \cdot I_M \cdot \dot{\varphi}_M^2 + \frac{1}{2} \cdot I_1 \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot I_2 \cdot \dot{\varphi}_2^2 + \frac{1}{2} \cdot I_3 \cdot \dot{\varphi}_3^2, \tag{3}$$

where  $I_M, I_1, I_2, I_3$  – moments of inertia of the wheels  $M, 1, 2, 3$ , respectively,  $\text{kg}\cdot\text{m}^2$ ;  
 $\dot{\varphi}_M, \dot{\varphi}_1, \dot{\varphi}_2, \dot{\varphi}_3$  – angular deflections of individual wheels  $M, 1, 2, 3$ ,  
 respectively,  $\text{m}\cdot\text{s}^{-1}$ .

As the system consists of rigid bodies, any relative motion occurs between individual pairs of the mechanical system. And, as it is mentioned above, the mathematical model will be reduced to the component  $M$ , i.e. to the wheel  $M$ . Therefore, all angular velocities ( $\varphi_1, \varphi_2, \varphi_3$ ) should be expressed by means of the angular deflection  $\varphi_M$  ( $\varphi_M$  is the generalized coordinate). This is possible to perform considering the gear ratios in the mechanism.

The gear ratio  $i_{M1}$  of the gear pair of the wheel  $M$  and the wheel 1 (Fig. 1) is:

$$i_{M1} = \frac{D_1}{d_M} = \frac{n_M}{n_1}, \quad (4)$$

Substituting the known parameters, we get the following relation:

$$i_{M1} = \frac{2 \cdot R_1}{2 \cdot r_M} = \frac{2 \cdot \pi \cdot \omega_M}{2 \cdot \pi \cdot \omega_1}, \quad (5)$$

where  $r_M, R_1$ , – radii of the wheel  $M$  and the wheel 1, respectively (Fig. 1), m;

$\omega_M, \omega_1$  – angular velocities of rotation of the wheel  $M$  and the wheel 1,  $\text{rad}\cdot\text{s}^{-1}$ .

Finally, we get the relation for the angular deflection  $\varphi_1$  as following:

$$\dot{\varphi}_1 = \frac{r_M}{R_1} \cdot \dot{\varphi}_M \Rightarrow \varphi_1 = \frac{r_M}{R_1} \cdot \varphi_M. \quad (6)$$

The gear ratio  $i_{12}$  of the gear pair of the wheel 1 and the wheel 2 is:

$$i_{12} = \frac{R_2}{r_1} = \frac{\varphi_1}{\varphi_2}. \quad (7)$$

When the relation (6) is considered, the resulting formulation is:

$$\varphi_2 = \frac{r_1}{R_2} \cdot \varphi_1 \Rightarrow \varphi_2 = \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot \varphi_M, \quad (8)$$

where  $r_1, R_2$ , – radii of the wheel 1 and the wheel 2, respectively (Fig. 1), m.

The gear ratio  $i_{23}$  of the gear pair of the wheel 2 and the wheel 3 is:

$$i_{23} = \frac{r_3}{r_2} = \frac{\varphi_2}{\varphi_3}. \quad (9)$$

and considering the relation (8), we get the formulation:

$$\varphi_3 = \frac{r_2}{r_3} \cdot \varphi_2 \Rightarrow \varphi_3 = \frac{r_2}{r_3} \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot \varphi_M. \quad (10)$$

The final mathematical expression of the kinetic energy is as follows:

$$D_K = \frac{1}{2} \cdot I_M \cdot \dot{\varphi}_M^2 + \frac{1}{2} \cdot I_1 \cdot \left( \frac{r_M}{R_1} \cdot \dot{\varphi}_M \right)^2 + \frac{1}{2} \cdot I_2 \cdot \left( \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot \dot{\varphi}_M \right)^2 + \frac{1}{2} \cdot I_3 \cdot \left( \frac{r_2}{r_3} \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot \dot{\varphi}_M \right)^2. \quad (11)$$

The further procedure requires partial derivations of this kinetic energy (according to the equation 1). As it is described above, these partial directions are performed according to the generalized coordinate  $\varphi_M$ . The required equation of motion is as following:

$$\left[ I_M + I_1 \cdot \frac{r_M^2}{R_1^2} + I_2 \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} + I_3 \cdot \frac{r_2^2}{r_3^2} \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} \right] \cdot \ddot{\varphi}_M = M_T. \quad (12)$$

The expression:

$$I_{red\_M} = \left[ I_M + I_1 \cdot \frac{r_M^2}{R_1^2} + I_2 \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} + I_3 \cdot \frac{r_2^2}{r_3^2} \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} \right] \quad (13)$$

is the reduced moment of inertia of the system (i.e. all wheels  $M$ , 1, 2 and 3, Fig. 1) to the wheel  $M$ .

The equation of motion (12) can be rewritten in a shorter form as following:

$$I_{red\_M} \cdot \ddot{\varphi}_M = M_T \cdot \quad (14)$$

where  $I_{red\_M}$  – moment of inertia of the mechanical system reduced to the component  $M$ ,  $\text{kg} \cdot \text{m}^2$ ;  
 $M_T$  – total torque acting in the system,  $\text{N} \cdot \text{m}$ .

In real mechanical drive system mechanisms, a belt is not rigid, however, it is flexible. It should be also considered in the mathematical model, because a belt is usually made of rubber or similar material with lower stiffness in comparison with other mechanical drive system components, i.e. wheels, or shafts and others. The mechanical drive mechanism with a flexible belt is shown in Fig. 2.

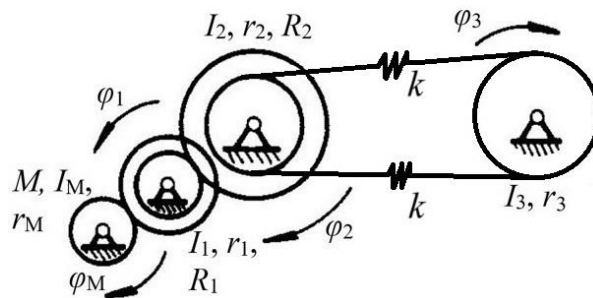


Fig. 2. Scheme of a dynamical model of a mechanical drive system with a flexible belt

As it is mentioned in Fig. 2, the belt flexibility is illustrated by a spring scheme and the belt stiffness  $k$  is considered in the mathematical model. This belt stiffness causes that the mechanical drive system has no longer one degree of freedom, however, it has two degrees of freedom due to the relative motion of the wheel 3 against the wheel 2. It should be mentioned that no friction as well as no slipping of the belt is considered. Moreover, the belt is still considered as massless. Hence, the considered relative coordinates will be the angular deflection  $\varphi_M$  and  $\varphi_3$ .

Thus, potential energy is accumulated in the flexible belt and its value is as following:

$$D_p = \frac{1}{2} \cdot k \cdot (r_2 \cdot \varphi_2 - r_3 \cdot \varphi_3)^2 + \frac{1}{2} \cdot k \cdot (r_2 \cdot \varphi_2 - r_3 \cdot \varphi_3)^2, \quad (15)$$

where  $k$  – torsion stiffness of the belt,  $\text{N} \cdot \text{m} \cdot \text{rad}^{-1}$ .

It means, that the mechanical drive system with the flexible belt will be described by means of two equations of motion. They are again derived by means of the Lagrange's equations of the second kind method (eq. 1). The angular motion will not be expressed only by means of the angular deflection  $\varphi_M$ , but also by means of the second generalized coordinate  $\varphi_3$ . Therefore, kinetic energy of the system with the flexible belt  $D_{K\_f}$  should be modified according to the scheme shown in Fig. 2. It is as following:

$$D_{K\_f} = \frac{1}{2} \cdot I_M \cdot \dot{\varphi}_M^2 + \frac{1}{2} \cdot I_1 \cdot \dot{\varphi}_1^2 + \frac{1}{2} \cdot I_2 \cdot \dot{\varphi}_2^2 + \frac{1}{2} \cdot I_3 \cdot \dot{\varphi}_3^2. \quad (16)$$

The inertial parameters of the wheel 1 and the wheel 2 will be again reduced to the wheel  $M$ .

Thus, the angular deflections  $\varphi_1$  and  $\varphi_2$  will be again expressed by equations (6) and (8). The final form of kinetic energy of the system with the flexible belt is:

$$D_{K\_f} = \frac{1}{2} \cdot I_M \cdot \dot{\varphi}_M^2 + \frac{1}{2} \cdot I_1 \cdot \left( \frac{r_M}{R_1} \cdot \dot{\varphi}_M \right)^2 + \frac{1}{2} \cdot I_2 \cdot \left( \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot \dot{\varphi}_M \right)^2 + \frac{1}{2} \cdot I_3 \cdot \dot{\varphi}_3^2. \quad (17)$$

The further procedure consists in partial derivations of the kinetic energy  $D_{K_f}$  and potential energy  $D_P$  according to the generalized coordinates  $\varphi_M$  and  $\varphi_3$ . The mechanical drive system is described by means of two equations of motion as following:

$$\begin{aligned} \left[ I_M + I_1 \cdot \frac{r_M^2}{R_1^2} + I_2 \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} \right] \cdot \ddot{\varphi}_M + 2 \cdot r_2^2 \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot k \cdot \varphi_M - 2 \cdot r_2 \cdot r_3 \cdot k \cdot \varphi_3 &= M \\ I_3 \cdot \ddot{\varphi}_3 - 2 \cdot r_2 \cdot r_3 \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} \cdot k \cdot \varphi_M + 2 \cdot r_3^2 \cdot k \cdot \varphi_3 &= 0 \end{aligned} \quad (18)$$

As it can be seen, that in this case the reduced moment of inertia of the system with the flexible belt  $I_{red\_M\_f}$  of the wheel  $M$ , the wheel 1 and the wheel 2 to the wheel  $M$  is:

$$I_{red\_M\_f} = \left[ I_M + I_1 \cdot \frac{r_M^2}{R_1^2} + I_2 \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} \right]. \quad (19)$$

The equations of motion (eq. 18) can be written in a matrix form as following:

$$\begin{bmatrix} I_M + I_1 \cdot \frac{r_M^2}{R_1^2} + I_2 \cdot \frac{r_1^2}{R_2^2} \cdot \frac{r_M^2}{R_1^2} & 0 \\ 0 & I_3 \end{bmatrix} \cdot \begin{bmatrix} \ddot{\varphi}_M \\ \ddot{\varphi}_3 \end{bmatrix} + \begin{bmatrix} 2 \cdot r_2^2 \cdot k \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} & -2 \cdot r_2 \cdot r_3 \cdot k \\ -2 \cdot r_2 \cdot r_3 \cdot k \cdot \frac{r_1}{R_2} \cdot \frac{r_M}{R_1} & 2 \cdot r_3^2 \cdot k \end{bmatrix} \cdot \begin{bmatrix} \varphi_M \\ \varphi_3 \end{bmatrix} = \begin{bmatrix} M \\ 0 \end{bmatrix}. \quad (20)$$

The derived equations (eq. 12 and eq. 20) are equations of motion of the mechanical drive systems with a rigid belt (eq. 12) and with a flexible belt (eq. 20). These equations represent time functions of output kinematic quantities (deflections, velocities and accelerations). They are solved by means of the computational software Matlab. The equations of motion (eq. 12 and eq. 20) are differential equations of the second order. The Matlab software allows to solve them, however, a modification of these equations to the differential equations of the first order is required. This process is not presented in this contribution.

## Results and discussion

This section includes the results of the simulation computation of the mathematical models of the mechanism calculated according to the derived equations of motion for the rigid belt (eq. 12) and with the flexible belt (eq. 18). The calculation of these equations of motion is performed by means of the Matlab software. As it is mentioned above, the equations of motion, i.e. differential equations of the second order should be modified to the differential equations of the first order. This is done by expression of accelerations of the systems by means of deflections and velocities.

Further, the solution of the equations of motion in Matlab assumes a definition of initial conditions. It means, that it is necessary to define the values of the initial angular deflections and velocities for the calculation. The calculation also needs to prescribe the time interval, in which these equations are solved. The time interval should be as long as the output variables are recognized.

The equations of motion, i.e. differential equations are solved in Matlab by means of the Runge-Kutta method [18-20]. It is a numerical method for solving simultaneous differential equations. It belongs to the iterative methods, which work with approximate solutions [18, 21-23].

In our case, the time interval of 5 seconds was chosen. The obtained results of the simulation computations are shown in Fig. 3 and Fig. 4.

Figure 3 depicts the waveforms of the output quantities for the mechanical system with a rigid belt. The outputs quantities are chosen as the drive torque  $M_1$ , the drive shaft revolutions  $n_1$  and the total load torque  $M_v$ . Figure 4 shows the results of the output quantities for the mechanical system with a flexible belt for the same time interval of 5 seconds. In this case, the output quantities the drive torque  $M_1$ , the drive shaft revolutions  $n_1$  and the total torque  $M_v$  are added by the output marked as  $\Delta_\phi$ . It is a relative deflection between the generalized coordinates  $\varphi_M$  and  $\varphi_3$ , i.e. relative deflection caused by the belt flexibility.

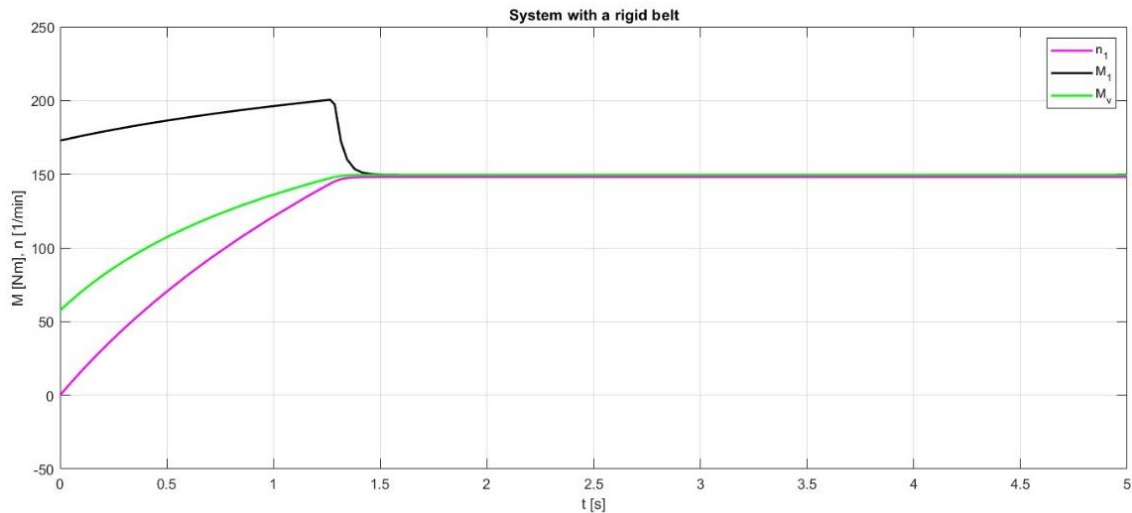


Fig. 3. Waveforms of the output quantities for the mechanical system with a rigid belt

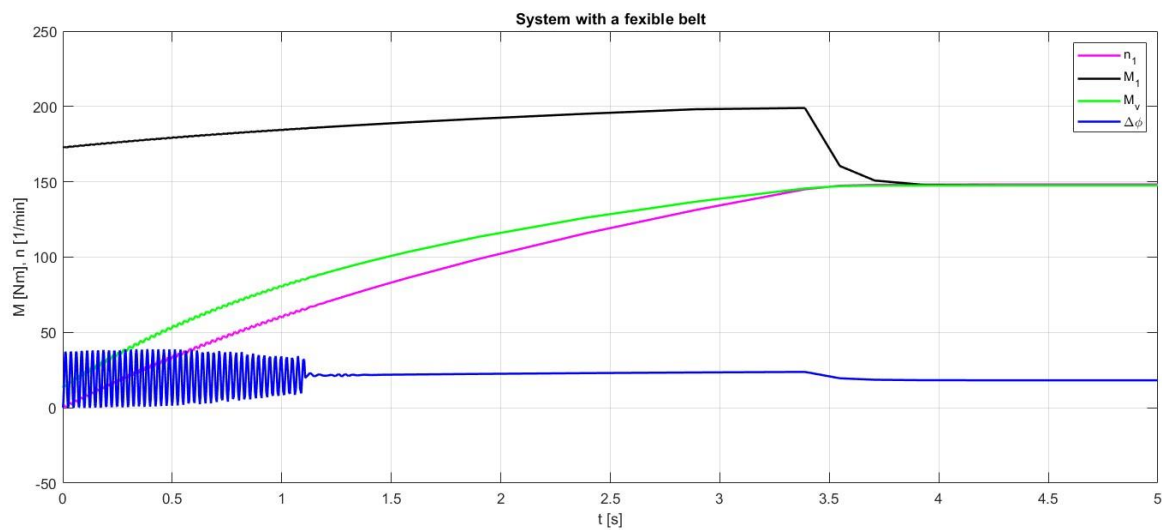


Fig. 4. Waveforms of the output quantities for the mechanical system with a flexible belt

The results and findings of the calculations are as follows. The drive torque  $M_1$  of the mechanism increases for both systems within a certain time interval until it equals to the total torque of the system  $M_v$ . While in the case of the rigid belt it takes approx. 1.5 seconds, in the case of the flexible belt it takes up to 4 seconds. Similarly, the same time intervals are needed for reaching the equilibrium for the revolutions of the drive shafts for both mechanisms, respectively. It is important to mention that the flexibility of the belt causes relative deflections (Fig. 4) of two parts of the mechanism, i.e. of the drive and driven parts. It is interesting, that the reaching equilibrium of the mechanism needs also a longer time interval in comparison with the mechanism with the rigid belt. These relative deflections of the drive and driven parts depend on the belt stiffness, the value of the drive torque, the value of the load torque and other parameters of the mechanical system of the mechanism [24-26].

The created model of the mechanical drive system also allows to input information in a parametric form for a combustion engine as well as for an electric motor. All matters about the modeling the input depends on the the requirements of the particular task. The presented model does not include irregularities during operation. However, the input data with irregularities can be considered. The Matlab software allows to create a model with a required waveform of the values, e.g. a waveform of the torque (for both a combustion engine and an electric motor or similar), a waveform of the loading torque (e.g. running resistances, variable loading depending on time) or other variable parameters, which can appear in the real operational conditions.

The future research will be focused on the creation of the investigated mechanism in the commercial multibody software. Simulations will be performed, and the results will be compared with the results presented in this contribution. When the results will be comparable within acceptable errors, it will be proven, that the equations of motion of the mechanism are derived properly and the process of the calculation can be also applied for other cases.

### Conclusions

1. The mechanical system of the mechanisms for drive of the additional accessories of transport means were presented and considered for the research. Two mechanisms were considered, namely with the rigid belt and with the flexible belt.
2. The mathematical models of both considered mechanisms were derived. The Lagrange's equations of the second kind were applied for derivation of the mathematical models. The system with the rigid belt is described by one equation of motion and the system with the flexible belt is described by two equations of motion.
3. The derived equations of motion were solved in the Matlab software for chosen time intervals. The results show that the flexibility of the belt affects the time, in which the system reaches the equilibrium. Further, the belt flexibility causes relative deflections of the drive and driven part of the mechanical system.
4. The solved tasks represent an example of the drive mechanism of additional accessories of a vehicle. The presented procedure of derivation of mathematical models can be also applied (after considered specific requirement of a particular task) for the study and investigation of other mechanical systems with rigid or flexible belts.

### Acknowledgements

This work was supported by the project KEGA No. 031ŽU-4/2023: "Development of the key competencies of the graduate of the study program Vehicles and Engines."

"Funded by the EU NextGenerationEU through the Recovery and Resilience Plan for Slovakia under the project No. 09I03-03-V01-00131."

### Author contributions

Conceptualization, J.D., A.L. and M.B.; methodology, J.D., A.L., M.B. and V.I.; software, J.D., A.L. and V.I.; validation, M.B. and D.M.; formal analysis, J.D. and A.L.; investigation, J.D., M.B. and V.I.; data curation, J.D., M.B. and D.M.; writing – original draft preparation, J.D. and A.L.; writing – review and editing, M.B. and D.M.; visualization, J.D. and V.I.; project administration, J.D.; funding acquisition, J.D. All authors have read and agreed to the published version of the manuscript.

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